Descriptive Approach to Image Analysis: Image Models

I. B. Gurevich and V.V. Yashina

Dorodnicyn Computing Center Russian Academy of Sciences, ul. Vavilova 40, Moscow, 119333 Russia

e-mail: igourevi@ccas.ru, werayashina@mail.ru

Abstract—The brief review of main methods and features of the descriptive approach to image analysis (DAIA), viz. forming the system of concepts that characterize the initial information–images–in recognition problems, and descriptive image models designed for recognition problems, is given.

At present, in terms of development of image analysis and recognition, it is critical to understand the nature of the initial information, viz. images, find methods of image representation and description to be used to construct image models designed for recognition problems, establish the mathematical language for the unified description of image models and their transformations that allow constructing image models and solving recognition problems, construct models to solve recognition problems in the form of standard algorithmic schemes that allow, in the general case, moving from the initial image to its model and from the model to the sought solution.

The DAIA gives a single conceptual structure that helps develop and implement these models and the mathematical language. The main DAIA purpose is to structure and standardize different methods, operations and representations used in image recognition and analysis. The DAIA provides the conceptual and mathematical basis for image mining, with its axiomatic and formal configurations giving the ways and tools to represent and describe images to be analyzed and evaluated.

DOI: 10.1134/S1054661808040020

INTRODUCTION

It is one of the breakthrough challenges for theoretical computer science to find automated ways to process, analyze, evaluate and understand information represented in the form of images. It is critical for computer science to develop this branch in terms of solving applied problems, in particular, increasing the diversity of classes of problems that can be solved and the efficiency of the process significantly.

Images are one of the main tools to represent and transfer information needed to automate the intellectual decision-making in many application areas.

Increasing the efficiency, including automatization, of gathering information from images can help increase the efficiency of intellectual decision-making. Recently, this part of image analysis called image mining in English publications has been often set off into a separate line of research.

We list the functions of particular aspects of image handling. Image processing and analysis provides for image mining, which is necessary for decision-making, while the very decision-making is done by methods of mathematical theory of pattern recognition. To link these two stages, the information gathered from the image after it is analyzed is transformed so that standard recognition algorithms could process it. Note that although this stage seems to have an "intermediate" character, it is the fundamental and necessary condition for the overall recognition to be feasible.

At present, automated image mining is the main strategic goal of fundamental research in image analysis, recognition and understanding and development of the proper information technology and algorithmic software systems. In the long run, this automatization is expected to help developers of automated systems designed to handle images as well as end users, either in the automated or interactive mode,

—develop, adapt and check methods and algorithms of image recognition, understanding and evaluation;

—choose optimal or suitable methods and algorithms of image recognition, understanding and evaluation;

—check the quality of initial data and whether they can be used in solving the image recognition problem;

—apply standard algorithmic schemes of image recognition, understanding, evaluation and search.

To ensure such automatization, we need to develop and evolve a new approach to analyzing and evaluating information represented in the form of images. To do it, the "Algebraic Approach" of Yu. I. Zhuravlev [43] was modified for the case when the initial information is represented in the form of images. The result is the descriptive approach to image analysis and understanding (DAIA) proposed and justified by I. B. Gurevich and developed by his pupils [9, 10, 14, 19].

By now, image analysis and evaluation have a wide experience gained in applying mathematical methods from different sections of mathematics, computer science and physics, in particular algebra, geometry, dis-

Received July 1, 2008

crete mathematics, mathematical logic, probability theory, mathematical statistics, mathematical analysis, mathematical theory of pattern recognition, digital signal processing, and optics.

On the other hand, with all this diversity of applied methods, we still need to have a regular basis to arrange and choose suitable methods of image analysis, represent, in an unified way, the processed data (images), meeting the requirements standard recognition algorithms impose on initial information, construct mathematical models of images designed for recognition problems, and, on the whole, establish the universal language for unified description of images and transformations over them.

In applied mathematics and computer science, constructing and applying mathematical and simulation models of objects and procedures used to transform them is the conventional method of standardization. It was largely the necessity to solve complex recognition problems and develop structural recognition methods and specialized image languages that generated the interest in formal descriptions–models of initial data–and formalization of descriptions of procedures of their transformation in the area of pattern recognition (and especially in image recognition in 1960s). As for the substantial achievements in this "descriptive" line of study, we mention publications by A. Rosenfeld [33], T. Evans [4, 5], R. Narasimhan [25–28], R. Kirsh [23], A. Shaw [35, 36], H. Barrow, A. Ambler, and R. Burstall [1], S. Kaneff [21]. In 1970s, Yu. I. Zhuravlev proposed the so called "Algebraic Approach to Recognition and Classification Problems" [43, 44], where he defined formalization methods for describing heuristic algorithms of pattern recognition and proposed the universal structure of recognition algorithms. In the same years, U. Grenander stated his "Pattern Theory" [8], where he considered methods of data representation and transformation in recognition problems in terms of regular combinatorial structures, leveraging algebraic and probabilistic apparatus. Both approaches dealt with the recognition problem in its classical statement and did not touch upon representation of initial data in the form of images.

Note that the idea to create a single theory that embraces different approaches and operations used in image and signal processing has a history of its own, with works of von Neumann continued by S. Unger, M. Duff, G. Matheron, G. Ritter, J. Serra, S. Sternberg and others [31, 32, 34, 37, 39] playing an important role in it.

Then, up to the middle of 1990s, there was a slight drop in the interest in descriptive and algebraic aspects in pattern recognition and image analysis.

By the middle of 1990s, it became obvious that for the development of image analysis and recognition, it is critical to:

(1) understand the nature of the initial information – images,

(2) find methods of image representation and description that allow constructing image models designed for recognition problems,

(3) establish the mathematical language designed for unified description of image models and their transformations that allow constructing image models and solving recognition problems, and

(4) construct models to solve recognition problems in the form of standard algorithmic schemes that allow, in the general case, moving from the initial image to its model and from the model to the sought solution.

The DAIA gives a single conceptual structure that helps develop and implement these models and the mathematical language [9, 10]. The main DAIA purpose is to structure and standardize different methods, operations and representations used in image recognition and analysis. The DAIA provides the conceptual and mathematical basis for image mining, with its axiomatic and formal configurations giving the ways and tools to represent and describe images to be analyzed and evaluated.

In this work, we give a brief review of the main DAIA methods and features, form of the system of concepts that characterize the initial information – images – in recognition problems and the descriptive image models designed for recognition problems. The work consists of three main sections (along with Introduction and Conclusions).

Section 1 "Descriptive Approach to Image Analysis and Understanding" states the DAIA mathematical basis and basic elements, including the model of the process of image recognition that is fundamental for the DAIA.

Section 2 "Images As Initial Data in the Recognition Problem" deals with the concepts needed to formally describe and represent images within the DAIA as well as forms and mathematical objects that reproduce the image in the course of constructing descriptive modes.

Section 3 "Descriptive Image Models" formalizes the ways of characterizing images, transformations and objects needed to describe images in the form that suitable for recognition algorithms. Finally, we consider the hierarchic scheme incorporating the concepts, definitions, mathematical objects and transformations that give the process and the result of constructing the image model in the recognition problem within the DAIA.

1. DESCRIPTIVE APPROACH TO IMAGE ANALYSIS AND UNDERSTANDING

To automate image mining, we need an integrated approach to leverage the potential of mathematical apparatus of the main lines in transforming and analyzing information represented in the form of images, viz. image processing, analysis, recognition and understanding.

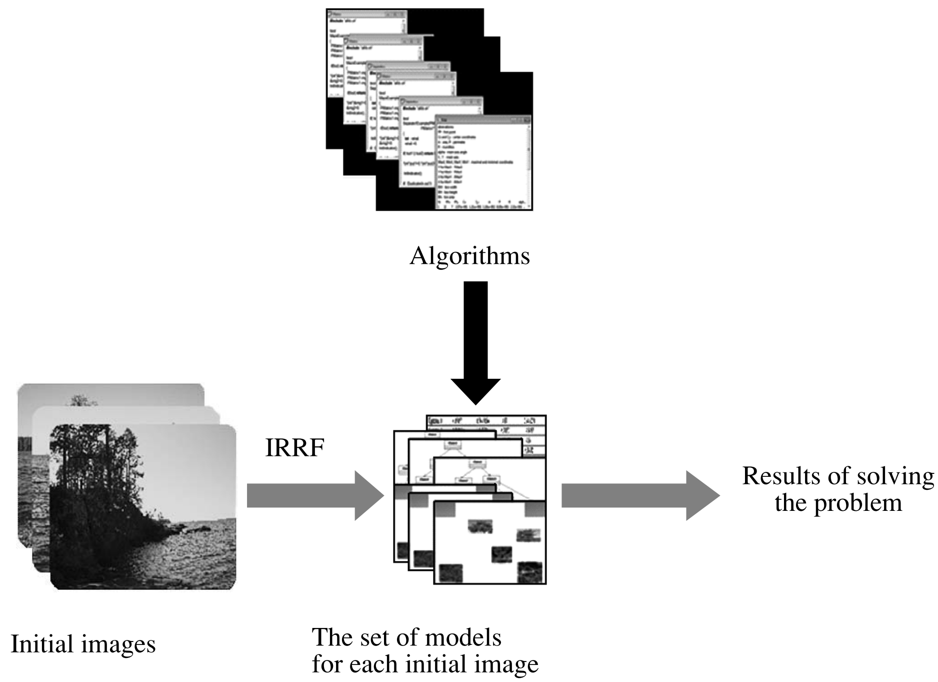


Fig. 1. Modern trends in image mining—multialgorithmic and multimodel. IRRF—Image reduction to a recognizable form.

Done by pattern recognition methods, image mining now tends to multiplicity (multialgorithmic and multimodel) and fusion of the results, i.e., several different algorithms are applied in parallel to process the same model and several different models of the same initial data to solve the problem and then the results are fused to obtain the most accurate solution (see Fig. 1).

Multialgorithmic classifiers [22, 42, 43] and multimodel and multiple-aspect image representations are the common tools to implement this multiplicity and fusion. Note that it was Yu. I. Zhuravlev who obtained the first and fundamental results in this area in 1970s [40–42].

Anticipating the more detailed discussion, note that the DAIA readily combines algorithms within multialgorithmic schemes and makes good use of multimodel and multiple-aspect representations in recognition problems. Still, the main problem is to reduce images to a recognizable form (IRRF—an image reduction to a recognizable form).

Proposed and being developed as a conceptual and logical basis for image analysis and recognition, the DAIA embraces a totality of methods of image analysis and recognition, methods of reducing images to a recognizable form, the system of concepts of image analysis and recognition, classes of descriptive image models (DIM), statements of image analysis and recognition problems, and the basic model of the image recognition process.

The main DAIA objects and tools are images, the universal language, viz. descriptive image algebras (DIA), and models of two types–image models and models of procedures to solve image recognition problems and their implementations in the form of algorithmic schemes.

Basically, within the DAIA, we:

(1) conceptualize and formalize the phases passed by the image when it is transformed in the course of solving the image recognition problem;

(2) classify and specify descriptive image models – DIM;

(3) use an algebraic language to describe image models and procedures to construct and transform them–DIA;

(4) implement image models and procedures to construct and transform them in the DIA language; and

(5) state image analysis and recognition problems and constructing the model of solving the standard image recognition problem.

From 1970s, the most part of image recognition applications and considerable part of research in artificial intelligence deal with images. As a result, new technical tools emerged to obtain information that allow representing recorded and accumulated data in the form of images and the image recognition itself became more popular as the powerful and efficient methodology to process and analyze data mathematically and detect hidden regularities. Various scientific and technical, economic and social factors make the application domain of image recognition experience grow constantly.

There are internal scientific problems that have arisen within image recognition. First of all, these imply algebraizing the image recognition theory, arranging image recognition algorithms, estimating the algorithmic complexity of the image recognition problem, automating the synthesis of the corresponding efficient procedures, formalizing the description of the image as the recognition object, making the choice of the system of representations of the image in the recognition process regular, and some others. It is these problems that form the basis of the mathematical agenda of the descriptive theory of image recognition developed using the ideas of the algebraic approach to recognition [43] to create a systematized set of methods and tools of data processing in image recognition and analysis problems.

There are three main issues one need to solve when dealing with images–describe (simulate) images; develop, study and optimize the selection of mathematical methods and tools of data processing in image recognition; and implement mathematical methods of image analysis on a software and hardware basis.

What makes image analysis and recognition problems peculiar, complex and thus difficult and catching is the necessity to find a compromise between rather contradictory factors. These factors are the requirements imposed on the analysis, the nature of visual perception, the ways to obtain, form and reproduce images and the existing mathematical and technical ways to process them. The main contradiction is between the nature of the image and the analysis based on formal description (a model, in essence) of the object, which lies in the fact that to leverage the fact that information is represented in the form of images, it is necessary to make this information non-depictive since the corresponding algorithms can only process certain symbolic descriptions.

Most methods of image processing are purely heuristic, with their quality essentially given by the degree to which they are successful in coping with the "depictive" nature of the image using the "non-depictive" tools, i.e., in employing procedures that do not depend on the fact that the information to be processed is organized in the form of images.

When we solve an image recognition problem, it is very important that we are able to choose the right recognition algorithm in a great number of known algorithms, i.e., we need to choose the best in some sense algorithm in the particular situation. It is obvious that both in image recognition and in solving recognition problems with standard teaching information [43], to make the choice of the best algorithm systematic, we need to introduce and formalize the corresponding objects of mathematical theory of image recognition, in particular, the concept of image recognition algorithm. It is known [43] that the necessity to state and solve the problem of choosing the algorithm extremal with respect to the recognition quality functional led to introducing the concept of the model of recognizing algorithm. To choose optimal or acceptable procedure to solve the particular problem, one needed to somehow fix the class of algorithms. This is the first reason that led to the necessity to synthesize models of recognition algorithms.

With the concept of the model of recognizing algorithm, we can apply strict mathematical methods to study the sets of incorrect recognition procedures (i.e., heuristic procedures that are not justified mathematically but were experimentally tested in solving real recognition problems). Analyzing the totality of incorrect recognition algorithms as they are accumulated, we can select and describe particular algorithms as well as principles to form them. Acting over subsets of algorithms and first formed in a poorly formalized form, these principles can then become accurate mathematical descriptions. At this stage, principles are chosen on a heuristic basis while algorithms generated according to it can be constructed in a standard way. It is in this sense that formalization of different principles of constructing recognizing algorithms results in models of recognizing algorithms.

To construct the model of recognizing algorithm, we need to describe sets of incorrect procedures that nevertheless are efficient in solving practical problems in a uniform way. To give such set, we specify variables, objects, functions, and parameters and their exact variation area, thus introducing the sought model of the algorithm. Given some set of the corresponding variables, objects, parameters and types of functions, we can single out some fixed algorithm from the model we consider.

To construct the model of an image recognition algorithm and determine the proper class of recognition algorithms, it is not enough to transfer the concept of the model of recognizing algorithm developed in the mathematical recognition theory [43] automatically to the image domain and directly use formal representations of a number of known recognition models studied in classical recognition theory [6, 43]. As noted above, the nature and matter of image recognition problems differ from that of the mathematical recognition theory in its classical statement.

When we move from classical recognition problems to image recognition problems, there arise mathematical problems due to formal description of the image as the object to be analyzed.

To obtain formal descriptions of images as objects to be analyzed and form and choose recognition procedures, we study the internal structure and content of the image as the result of the operations that can be performed to construct it of sub-images and other objects of simpler nature, i.e., primitives and objects singled out on the image during different stages of handling it (depending on the aspect, morphological and/or scale level used to form the image model). Since this way of characterizing the image is operational, we can consider the whole process of image processing and recognition, including construction of formal description –

model of the image, as a system of transformations implemented on the image and given on the equivalence classes that represent ensembles of admissible images [12, 13]. Hence, we operate with the hierarchy of formal descriptions of images, i.e., image models used in recognition relate to different aspects and/or morphological (scale) levels of image representation. In essence, these are multiple-aspect and/or multilevel models that allow choosing and changing the necessary degree of detail of description of the recognition object in the course of solving the problem. This approach to formal description of images forms the basis for the multimodel representation of images in recognition problems.

A recognition problem with standard information is to calculate the values of predicates $P_i(\omega)$—"$\omega \in \Omega_i$," $i = 1, \ldots, m$ for the given object ω and set of classes $\Omega_1, \ldots, \Omega_m$, using the teaching information $I_1(\Omega_1, \ldots, \Omega_m)$ on classes and the description $I(\omega)$. Symbols "1" ($\omega \in \Omega_i$), "0" ($\omega \notin \Omega_i$), and Δ (it is not known if ω belongs to the class Ω_i) encode the occurrence of the object ω in the class Ω_i, which is written in the form of the so called information vector

$$\tilde{\alpha}(\omega) = (\alpha_1(\omega), \ldots, \alpha_m(\omega)), \quad \alpha_i \in \{0, 1, \Delta\}. \quad (1.1)$$

We call standard information

$$I_1(\Omega_1, \ldots, \Omega_m) \quad (1.2)$$

the totality of sets $(I(\omega_1), \ldots, I(\omega_{r_m}))$ and $(\tilde{\alpha}(\omega_1), \ldots, \tilde{\alpha}(\omega_{r_m}))$ (it is assumed that there is no vector of the form $(\Delta, \ldots, \Delta)$ among information vectors). We consider objects $\omega_1, \ldots, \omega_{r_1}$ to belong to the class Ω_1, objects $\omega_{r_{i-1}+1}, \ldots, \omega_{r_i}$ to belong to the class Ω_i, and objects $\omega_{r_{m-1}+1}, \ldots, \omega_{r_m}$ to belong to the class Ω_m.

Thus, any recognition algorithm (in the classical case) transforms the recognition problem z with q objects to be recognized and m classes into the matrix of responses $\|\alpha_{ji}\|_{q \times m}$, viz. the information matrix the rows of which are information vectors (1.1) for each object ω^j, $j = 1, \ldots, q$ to be recognized. The recognizing algorithm processes the initial information into the information matrix of responses in two sequential stages. In the first stage, the initial information is translated into some numerical matrix $\|a_{ji}\|_{q \times m}$ of standard size with the number of rows equaling the number of objects to be recognized in the problem z and the number of columns equaling the number of classes we consider when solving the problem z

$$B(I, \Omega_m, \omega_j) = \|\alpha_{ji}\|_{q \times m}. \quad (1.3)$$

At its second stage, the recognizing algorithm transforms this numerical matrix into the matrix of final responses $\|\alpha_{ji}\|_{q \times m}$ with the same number of rows and columns

$$C(\|a_{ji}\|_{q \times m}) = \|\alpha_{ij}\|_{q \times m}. \quad (1.4)$$

This means that, for the pattern recognition problem in its classical statement, we can represent any recognition algorithm as two sequentially performed algorithms [43]. The first algorithm is the recognizing operator B that performs the main part of information processing while the second algorithm is the decision rule C that, in principle, can be similar for all recognizing algorithms.

When operating with such forms of information as symbolic and numerical (observation, measurement, examination results, numerical characteristics, parameters, expert evaluations, etc.), we can formalize recognition problems using standard information (1.2) and matrices of form (1.3) and (1.4) as it is. In these cases, formal description of recognition objects—models—are relatively simple with minimal requirements imposed on arranging and representing the initial information; it is quite admissible to use simple tables of the "attribute—attribute value" type [6]. The main feature of such models of recognition objects is that they are represented by a set of characteristics, the interconnection and relationships between which are not taken into account. Each object is assumed to be identified with some point of the multidimensional attribute space, the class of objects being represented by a "compact" set of such points.

The analysis held allows us to state the main features of image recognition problems.

(1) Constructing the formal description – the model of the image as the recognition object – becomes an independent problem solved in the recognition process.

(2) The model of the image should include structural or at least relational information, i.e., it should represent a formal configuration obtained, taking into account the hierarchy of the structure of the recognition object and relations that exist between individual elements of this hierarchy both within the same levels and between them.

(3) Concepts of initial and final information change. Image recognition is multilevel, with the initial model of the image transformed via the recognition procedure into the image model associated with another aspect or morphological (scale) level. The recognition procedure is applied to the obtained model and so on, the rule of stopping given by the form of the result to be obtained in the course of solving the problem in hand.

(4) The fact that image description and recognition are closely connected and that we need to include image models associated with different aspects and/or morphological (scale) levels in the iterative recognition process means that the image recognition algorithm, in addition to the recognizing operator and decision rule,

includes the operator that reduces the image to an easy-to-recognize form

$$R_f^n(I_n) = P_n(I, \Omega_m, \omega_j), \quad n = 1, \ldots, t, \tag{1.5}$$

where I_n is the image corresponding to some aspect and/or morphological (scale) level of description n; $P_n(I, \Omega^m, \omega^j)$ is the image model in the n-th aspect and/or at the n-th morphological (scale) level of the description that was obtained by applying the operator R_f^n that reduces the image to an easy-to-recognize form. Recognizing operator B (1.3) is naturally applied to the model P_n, and the result can be another image model P_{n+m} associated with some "initial" aspect or "lower" morphological (scale) level.

Having listed the features of image recognition problems, we can give the class of recognition procedures characterized by a fixed structure given by sequential application of the triples of operations R_f^n (1.5), B (1.3), and C (1.4). The interpretation of the procedure, i.e., particular forms of transformations $\{T^F\}$ and $\{T^A\}$ that process information in the course of recognition are given by the purposes and type of the analysis carried on

$$T^F: I_n \longrightarrow P_n(I^*); \tag{1.6}$$

$$T^A: P_n(I^*) \longrightarrow I^R, \tag{1.7}$$

where $P_n(I^*)$ is the model of the observable image that corresponds to the aspect or morphological (scale) level n; I^R is the regularity found on the image in the course of solving the recognition problem [6].

On the whole, whether analysis and evaluation of information represented in the form of images is successful and efficient depends on ETRR capabilities. ETRR processes are critical in solving applied problems of image analysis and, in particular, in intellectual decision-making based on image mining. ETRR main problems and open issues imply:

(1) constructing formal description of images:

(a) studying and constructing image models;

(b) studying and constructing multimodel representation of images;

(2) describing classes of images that can be reduced to a recognizable form:

(a) giving new mathematical statements of image recognition problems;

(b) identifying and studying relations between multimodel representations of images and image metrics;

(c) studying and using image equivalence;

(3) developing, studying and applying the algebraic language to describe ETRR procedures.

We can leverage the DAIA to solve both problems of constructing formal descriptions of images as recognition objects and problems of synthesizing image recognition and understanding procedures. Within the operational approach to characterizing images, we can consider processes of analyzing and evaluating information represented in the form of images (including synthesizing the formal description of the image, analysis, evaluation and recognition), viz. trajectories of solving problems, as a sequence/combination of transformations and obtaining some set of intermediate and final (that give the solution) estimates. We specify these transformations on classes of image equivalence. The classes are given descriptively, using the basic set of prototypes and their corresponding generating transformations that are fully functional with respect to the class of equivalence of admissible transformations.

Thus, the processes of analyzing and evaluating information given in the form of images are trajectories formed by admissible transformations in the space of formal descriptions of images. This space is of dual nature since it consists of both objects and the results of transformations specified. It is hierarchic and includes models of different types. These models can correspond to different morphological and scale levels of image representation, represent different aspects of image properties and characteristics and include multilevel, multiple-aspect and multimodel image descriptions. This allows us to choose and vary the degree of completeness and aspects of description of the image when solving the particular problem.

A formal description of the image is given by the set of objects selected on it, connected by structural relations and constructed by admissible generating transformations. Using the generating principles, bases of transformations and bases of models, we can divide problems into primitive subproblems, establish the correspondence between basic primitive problems and basic primitive transformations and combine basic algorithms and models.

Image mining employs different types of knowledge, viz.:

knowledge on subject domain;

knowledge of the nature and specifics of the problem;

knowledge on physical and geometric aspects of the scene to be analyzed;

logical, mathematical and physical laws obeyed by the scene to be analyzed;

knowledge on the ways and tools to obtain, register and form the image;

etc.

This knowledge is applied to construct image models (choose primitive elements and characteristic objects of the image, aspects, ways and levels of its formalization), models of recognition processes and their control models (form hypotheses on possible results, choose heuristic transformations; the rule of stopping). Generally, knowledge used is limited to context and semantic information and sets of logical and physical conditions.

To sum it up briefly, note that when solving recognition problems within the DAIA, data processing and representation procedures are arranged in a standard way. The recognition process is multilevel and multiple-aspect in its structure. For each level (aspect), we choose, find and calculate attributes used to construct the image model that corresponds to this level (aspect). For attributes, we use statistical, topological, geometric, structural and spectral characteristics of the image and its local fragments (neighborhoods), generalized attributes-objects singled out on the image, procedures-attributes that find whether it is possible to determine standard systems of transformations on the image, and attributes that characterize the results of applying standard systems of transformations to the image and its local neighborhoods (skeletons, medial axes). To choose attributes, we use knowledge on the subject domain and logical and physical restrictions characteristic of the scene given on the image. To synthesize the model within each aspect (level), the method of reverse algebraic closure (see Section 1.2) is applied. To move from the lower-level model (“initial sketch” [24]) to upper-level models and, finally, to the sought description that act as the solution to the recognition problem, we perform transformations included in the structure of the reverse algebraic closure as well. The recognition process is structured both horizontally (for image model construction) and vertically (for the recognition itself) in the sense that each iteration is implemented as a reverse algebraic closure.

The DAIA is based on:

(a) the descriptive model of the image recognition problem;

(b) special mathematical statements of image analysis and recognition problems (all operations of image analysis and recognition are written by means of specialized algebras);

(c) images reduced to an easy-to-recognize form (the mandatory step of the image recognition problem that takes into account depictive properties of the initial information and formal requirements on data that go directly to the input of recognition algorithms);

(d) algebraization of image mining (specialized versions of image algebras);

(e) the generating principle and basic transformations and models (the description of the image is constructed as a hierarchic structure formed by simpler objects);

(f) multiplicity of image models – multimodel and multiple-aspect image representations (being able to choose and change the necessary degree of detail of description of the recognition object in the course of recognition process, i.e., use “partial” and specialized “aspect” models); and

(g) introduction of knowledge into the processes of image mining.

1.1. Main Problems

Image processing is to prepare images to analysis and recognition, i.e., remove noise, improve the quality, remove “unnecessary” details and image fragments, detect objects and select their contours (if possible at this stage), perform statistical and logical filtration, and single out some attributes of images.

Image analysis is to perform the main tasks of gathering information from images that can be used to make intellectual decisions regarding objects, situations and scenes given on the image. The most important result of image analysis is IRRF, i.e., constructing the formal description – the image model.

Image recognition implies stating and solving recognition problems, with the initial information given in the form of individual images, multiple images and image models. Recognition results in attributing the image to be recognized, its fragments or individual objects given on the image to some class or dividing the objects to be recognized into non-overlapping subsets (classes). Thus, this is the stage when intellectual decisions are generally made.

Image understanding simulates functionality of the human visual system, in particular, gathering knowledge on three-dimensional world from two-dimensional images and using the two-dimensional image or set of images to describe the three-dimensional scene given on the image (images). Image understanding leads to the symbolic description of the image in the language of its elements, relations between them and image properties. This description should be suitable for decision-making in the real three-dimensional world (three-dimensional object recognition, automatic navigation). To implement image understanding, the results of processing, analysis and recognition are combined with the knowledge (context and general) on the scene shown.

Thus, all listed processes of image transformation are reduced to solving standard problems. However, these standard problems are highly diverse. Singling out standard problems increases the potential of both image mining and its automatization significantly since each of these problems and some of their combinations can be put into correspondence with standard algorithmic schemes. To standardize these problems, we need to divide them with respect to several levels (aspects) – subject problems; problems singled out in the general model of image mining; and technical problems.

1.1.1. Subject problems. These problems correspond to main types of decisions made using information represented in the form of images. It is not difficult to see that these are generated by standard statements of the recognition problem as applied to the cases when information is given in the form of images. The problems are to

(1) compare two images in whole to find whether they belong to one class (to determine whether the images represent the same object or scene);

(2) compare the image in whole with the set or series of sequential (in time) images that represent some class of images (i.e., objects or scenes) (the purpose is the same as in problem 1);

(3) problems 1 and 2 for several classes;

(4) on the image to be recognized, find some regularity/non-regularity, object or "situation" that deserves attention though it was not given in the a priori list of etalons (associative search; boundedly deterministic set of classes, viz. problems of logical and semantic filtration combined with self-learning);

(5) on the image to be recognized, find some regularity/non-regularity, fragment, object or "situation" of the given form;

(6) divide the set of images into non-overlapping subsets (the problem of automatic classification);

(7) solve the problem of automatic classification on one image (divide the image into homogeneous areas, groups of pixels and objects, segment the area, single out attributes of objects);

(8) solve problems 6 and 7 together;

(9) automatically single out primitive elements, characteristic objects of the image, attributes-objects, spatial and logical relations to synthesize formalized descriptions of the image;

(10) reduce the image to an easy-to-recognize form; synthesize formalized descriptions of the image automatically;

(11) problems of restoring:

missed frames in the sequence of images;

the whole images by their fragments;

fragments of the image (and objects), using primitive elements, attributes and generating procedures and taking into account the overall context of the image; and

the trajectory of the problem by its fragments and unknown fragments of trajectories;

(12) decompose the problem of image processing and analysis into elementary basic subproblems;

(13) choose and form the trajectory of the image recognition problem (in the sense of the recognition problem with standard teaching information); and

(14) solve problems 1–13 when there are dynamic objects and complex background (including dynamic and statistical noise) on images and taking into account the way images are obtained, formed and represented.

1.1.2. Functional problems. We associate the second aspect of classification of image transformation problems with dividing the image mining process into components according to the purpose (function) of its stage we single out. The main problems of this functional aspect of classification are to:

(1) process images;

(2) analyze images:

(a) segment the image (divide the image into non-overlapping fragments);

(b) measure the objects and characteristics on the image;

(c) single out attributes that describe the structure and content of the image;

(d) search, single out and construct individual objects that show physical real-world objects;

(e) establish interdependences and single out relations between objects of the image;

(f) calculate values of characteristics used to construct the image model; and

(g) synthesize the model;

(3) recognize and understand images:

(a) construct the consistent description of the scene that makes it interpretable;

(b) construct the three-dimensional model;

(c) classify the objects singled out in accordance with problems of decision-making and knowledge on the subject domain; and

(d) make intellectual decisions.

1.1.3. Technical problems. Primary image processing problems that perform pixel-by-pixel processing and measure characteristics of fragments, localize objects and characteristic elements of images that ensure gathering information needed for image mining are to:

(1) single out significant groups of pixels and objects on the two-dimensional image by analyzing their location, spatial relations and brightness and time characteristics;

(2) construct and single out surfaces, volumes, boundaries, shadows, occlusions, depth, color and motion;

(3) perform affine transformations over images (reflection, rotation, shift, re-proportioning, and scaling; transform the image from the observer's coordinates into the real-world coordinates);

(4) perform binarization and ranging of grey level images;

(5) improve the image quality (smoothing, adding or removing noise, retouching, filtration, sharpening, improving the texture, aligning image histograms; adding special effects into images);

(6) perform topological transformations (labeling linked components, image thinning);

(7) perform geometric transformations (the Delaunay triangulation, the Voronoi diagram, Haff transformations);

(8) perform operations of signal transformation (the Fourier transformation, obtaining image spectrum, Walsh transform); and

(9) solve primary problems of image restoring.

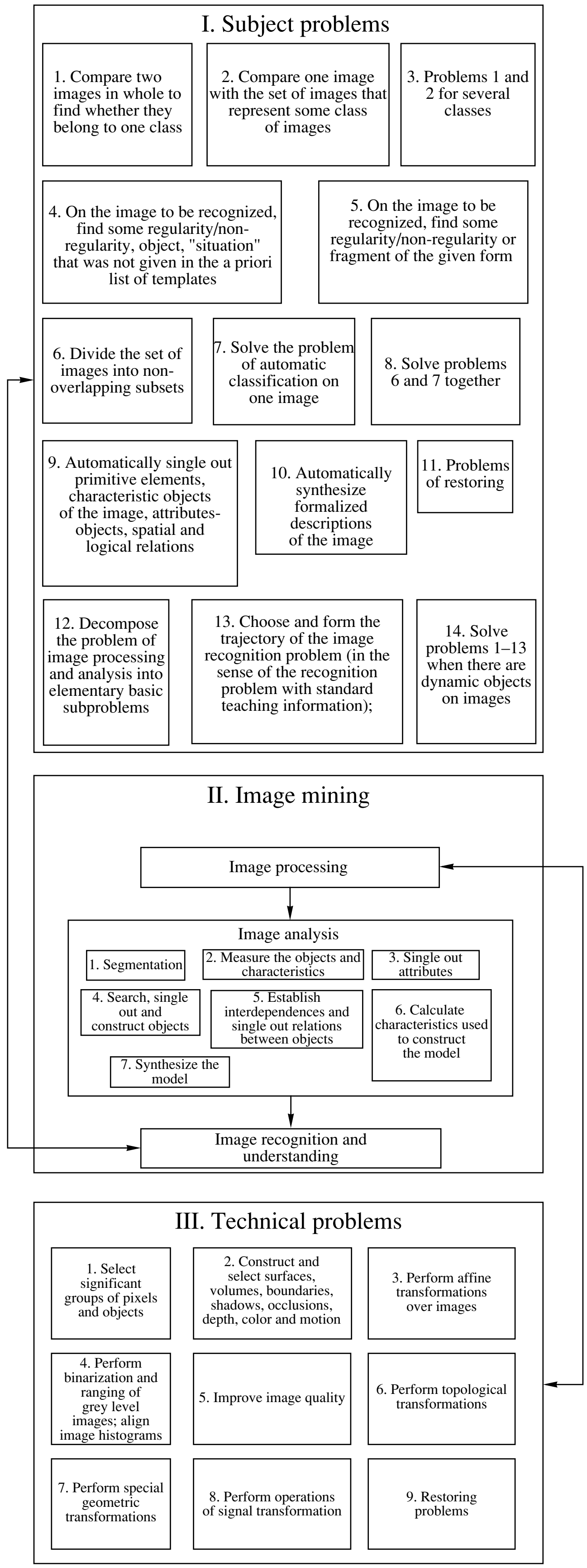
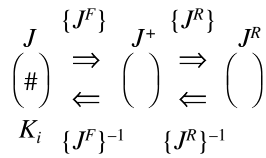


Fig. 2. Three aspects of classification of problems.



$$J, J^+, J^R; T^F, T^R, T^F : J \Rightarrow J^+, T^R : J^+ \Rightarrow J^R; (J) = \bigcup_j K_j.$$

Fig. 3. The descriptive model of the image recognition problem.

Figure 2 illustrates these three aspects of classifying standard image processing, analysis, recognition and understanding problems.

1.2. Descriptive Model for the Image Recognition Problem

Image analysis and recognition deal with properties of the object (scene) shown and deformations associated with the way and procedure of obtaining the image. In this case, to formalize image processing, we need to specify three sets (models) of images, on which we postulate the existence of classes of equivalence and sets of admissible transformations given on the classes of equivalence [12, 13]. Introducing classes of equivalence on the sets of image models, we accept that any image possesses some regularity or a mix of regularities of different types. Under this assumption, analysis and recognition problem is reduced to making a difference between images that preserve their own regularity and images, the regularity of which can be broken. (It is natural that there can be problems stated to search the image for regularity or irregularity of certain types.)

Figure 3 shows the descriptive model of the image recognition problem.

Here, $\{J\}$ is the set of ideal images, $\{J^*\}$ is the set of observable images, $\{J^R\}$ is the set of images obtained as a result of solving the recognition problem, $\{T^F\}$ is the set of admissible transformations to form the image, $\{T^R\}$ is the set of admissible transformations to recognize the image, and $\{K_i\}$ are classes of equivalence.

Let J be some true image of the object involved. We can consider processes of obtaining, forming, discretization, etc. (all procedures that make it possible to work with the image) as if the true image were transferred via the noisy channel. As a result, we analyze some real (observable) image J^* rather than the true image. This real image is to be classified in the course of analysis, i.e., we should determine the prototype in the true class of equivalence or find the regularity (regularities) of the given type J^R on the observable image J^*.

Thus, we can specify the sets $\{J\}$, $\{J^*\}$, and $\{J^R\}$ and transformations to form (T^F) and recognize (T^A) the image

$$T^F: J \longrightarrow J^*, \quad (1.8)$$

$$T^A: J^* \longrightarrow J^R. \quad (1.9)$$

To perform image recognition, we need to give algebraic systems of transformations $\{T^F\}$ and $\{T^A\}$ on classes of equivalence of the set $\{J\}$ and apply them to observable images J^* to perform the backward analysis, i.e., classify images according to the nature of their regularity (restore true images, i.e., indicate classes of equivalence they belong to), and the forward analysis, i.e., search the image J^* for regularities of the certain type J^R and localize them.

Stating the analysis problem in such a way, we can give the class of image processing procedures, analysis process of which is of fixed structure, with interpretation (particular implementation) depending on the purposes and type of analysis. There are following main stages of analysis.

(a) Synthesizing models of the observable image. We can use any type of model introduced in the DAIA (see Section 3). Local attributes help bring information on the image structure into the model. We give the model by the generalized inductive definition, using combinatorial regular structures. If the model chosen is parametric, the set of parameters that characterize global and local properties of the image is used to encode it. The main challenges in synthesizing such models are connected with the encoding efficiency, which depends on the ratio between the global and local information and the extent to which the structural information is used. There are two ways to determine global characteristics of the image—by local characteristics or using the overall image directly.

For local characteristics, it is reasonable to use characteristics based on:

(1) the Shannon measure calculated for the distribution of types of neighborhoods of individual image elements;

(2) the distribution of types of Boolean functions given on the neighborhoods of the image elements;

(3) the distribution of types of partially defined Boolean functions given on the neighborhoods of the image elements; and

(4) numerical estimates of properties of connectivity graphs and partially defined connectivity graphs of homogeneous parts of the neighborhoods of the image elements.

For global characteristics, it is reasonable to use those calculated when integer matrices are folded (for instance, permanents).

(b) Performing logical filtration of the image. This stage implies the preconditioning of the observable image so that it could be classified preliminarily, which is necessary to choose the set of transformations $\{T^F\}^{-1}$. We assume that there is a correspondence between the type and/or character of the model $R(J^*)$ and the class of equivalence given on the set $\{T^F\}$.

(c) Finding the class of equivalence (true) for *R*(*J).** To do this, we apply the procedures given by the linear closure with respect to the set of transformations $\{T^F\}^{-1}$ to the model $R(J^*)$ (the "backward" closure)

$$L\{T^F\}^{-1}: R(J^*) \Rightarrow K[R(J^*)]. \tag{1.10}$$

(d) Choosing the set of transformations $\{T^A\}$ to analyze *R*(*J).**

(e) Applying the procedures to the model *R*(*J).**

The procedures are given by the linear closure with respect to the set of transformations $\{T^A\}$ (the "forward" closure) in order to find the sought regularities J^A

$$L\{T^A\}: R(J^*) \Rightarrow J^R. \tag{1.11}$$

(f) Using the "feedback." We compare the found regularities J^{R*} with the regularities inherent to the class of equivalence of the true image $\{J^K\}$. To do this, we apply the "backward" closure procedure to the found regularities and the "forward" closure procedure to the regularities of the true image

$$L\{T^A\}^{-1}: J^R \Rightarrow J^{R*}, \tag{1.12}$$

$$L\{T^F\}: J^K \Rightarrow J^{K*}. \tag{1.13}$$

(g) End of the process. When J^{R*} coincides with J^{K*}, the analysis stops; otherwise, it repeats, using algebraic instead of linear closures in procedures (c) and (e). Corrective operations can be also applied.

As a whole, the described procedure is called a reversible algebraic closure.

Introducing the descriptive model of image recognition leads to special mathematical statements for the problem of image analysis and recognition.

1.3. DAIA Mathematical Aspects

With the descriptive model of the image recognition problem given by the reversible algebraic closure scheme, the sets of analysis transformations and sets of information matrices obtained as the result of their application form certain algebras. Mathematical problems related to this model involve studying the properties of these algebras and is close to the algebraic approach to recognition [12] (in particular, it is constructing correct models for the corresponding classes of recognizing operators (transformations), the stability of the correct algorithm within closures, constructing the correct algorithm within the reversible algebraic closure efficiently).

As for the mathematical basis for automated image mining, the DAIA suggests:

specializing Zhuravlev's algebras for the case when information is represented in the form of images;

standardizing representations of image analysis and recognition problems;

standardizing the language used to describe image analysis and recognition procedures; and

using the standard mathematical apparatus to perform operations over image analysis and recognition algorithms and over image models.

To develop the sought mathematical theory, we apply the algebraic approach to recognition and classification problems (Zhuravlev, academician, [43]) to introduce algebraization to the theory, modify the approach to the case when information is represented in the form of images (DAIA, Gurevich [9, 10, 14, 19]) and develop image algebras and descriptive image algebras.

DIA [11, 16], descriptive image models (DIM) [11, 19] and multimodel and multiple-aspect representations of images based on generating descriptive trees (GDT) [15] are the main DAIA tools.

As shown by the attempts made to create it, the formal apparatus to represent image processing and analysis procedures in a uniform and compact form should be based on a formal system of image representation and transformation that meets the following conditions:

(a) points, sets, models, transformations, and morphisms can be used as objects;

(b) each object is a hierarchic structure constructed of primitive objects by some transformations; and

(c) each transformation is a hierarchic structure constructed of basic transformations by some transformations.

This formal system (which is essentially a formal language and formalisms based on it) should allow and

—constructing formal configurations (for instance, algebraic structures) that make it possible to apply methods from different branches of mathematics and computer science in image processing, analysis and recognition;

—constructing accurate and compact image descriptions handy in terms of both the way to interpret the actions performed and the development of new methods;

—describing transformations over images in the form of compact sets of simple transformations both in the machine-independent form and in the form adapted to particular architectures;

—creating specialized sub-languages to describe images and transformations over them in certain classes of image recognition and evaluation problems;

—increasing the efficiency of software implementation; and

Basic DIA variants

Elements of the ring	Operations of the ring
(1) Operations for calculating numerical attributes	(1) Standard algebraic operations
	(2) Image algebra operations
(2) Complex algebraic operations	(1) Standard algebraic operations
	(2) Image algebra operations on the subset of operations
	(3) Complex algebraic operations
(3) Standard algebraic operations with parameter	(1) Standard algebraic operations
	(2) Image algebra operations
	(3) Complex algebraic operations
(4) Images	(1) Standard algebraic operations
	(2) Image algebra operations
	(3) Complex algebraic operations
(5) Images and their representations	Complex algebraic operations

—choosing the most efficient programming languages in terms of formal structures that describe known algorithms of image processing, analysis and recognition.

Having analyzed the requirements to its functionality and mathematical characteristics, we can see that the sought formal system should represent a certain, special class of algebras that makes it possible to write any image transformation algorithm as a combination of elementary basic operations. Thus, this class of algebras should allow handling both main image models—analysis and recognition objects—and main models of transformations that allow synthesizing and implementing basic procedures of formal image description, processing, analysis and recognition efficiently.

The DAIA defines a new class of image algebras, viz. descriptive image algebras (DIA), as the sought algebraic language to describe, compare and standardize image analysis, processing and recognition algorithms. With these algebras, we can combine and standardize procedures of processing image models and their transformations.

DIA makes the process of constructing and applying algorithmic schemes of image mining flexible and standardized. To give problems, objects and transformations associated with image mining, we use hierarchic structures that result from applying DIA operations to the set of primitive problems, primitive elements of the image and basic transformations. Within such approach, we can vary methods of solving the subproblem – use operations of image analysis as DIA elements, keeping the overall image mining technique unchanged.

We recall the main definitions of image algebras.

Definition 1.1 [37]. *A Sternberg image algebra* is a representation of image processing algorithms of the cell computer in the form of algebraic expressions, with images serving as their variables and procedures of constructing logical or geometric image combinations, as their operations.

Definition 1.2 [31]. *A standard Ritter image algebra* is a heterogeneous or multivalued algebra of complex structure of operands and operations if images (the set of points) as well as their related values and characteristics (the set of values related to the points) are the main operands.

Definition 1.3 [16]. An algebra is called a *descriptive image algebra* if its operands are either image models (both the image itself and the set of its related values and characteristics can be chosen as a model) or operations over images or both.

Definition 1.4 [16]. The ring U, which is the finite-dimensional vector space over some field P, is a *descriptive image algebra with one ring* if its operands are either image models or operations over images.

What makes DIA special is that:

(1) by imposing restrictions on basic DIA operations, new mathematical constructions (DIA) ensure that we use the concept of algebra in its strict classical sense and

(2) basic DIA operations are introduced both over images and over arbitrary formal representations of images as well as over image transformations.

The latter explains why this new type of algebras has the word "descriptive," viz. dealing with image descriptions, in its name. Using the concept of algebra in its strict classical sense in the DIA definition, we can single out basic DIA operations for different types of operands, thus having the set of complete systems to describe image analysis problems.

Studying the new class of algebras, we [11, 14, 16, 20]:

—defined DIA, basic DIA and DIA with one ring and propose the hierarchic classification of modern algebras, specifying the place of DIA in it;

—proposed the method of checking properties of operands and operations to construct DIA with one ring (this method takes the specifics of new image algebras into account);

—singled out operations of the standard image algebra [31] to be used to construct DIA (we solved this problem when studying operations of DIA with one ring);

—proposed the ways to construct DIM using special classes of DIA (which is necessary to formalize algorithmic schemes of image analysis); and

—found necessary and sufficient conditions for DIA with one ring to be generated (which were obtained when studying the main DIA operands and operations).

The table gives the examples of basic DIA variants.

For more detailed information on DIA, see [11, 14, 16, 20].

2. IMAGE DESCRIPTION FORMALIZATION

In this section, we form the system of concepts that characterize the initial information, viz. the image, in recognition problems. The system of concepts we introduced provides the basis for formal definition of methods of synthesizing image models and descriptive image models designed for image analysis and recognition problems.

2.1. Images as Initial Data in the Recognition Problem

To develop methods of automating image recognition, we need to find efficient ways to formalize images so that to reflect image semantics, information hidden in its internal structure and the structure of external bonds of part of the real world (scene) reproduced by the image. So far, there are no systematic mathematical methods of image formalization and analysis. The overwhelming majority of methods of image handling are heuristic, their advantages depending on the way they use "non-depictive" tools to convey the "depictive" features of the image.

By its nature, the image is an object of complex information structure that reproduces information on the initial scene, using values of brightness of discrete elements of the image, viz. pixels, patterns of image fragments, sets of pixels and spatial and logical relations between patterns, sets of pixels and individual pixels. What make images different from other tools of data representation is that they are highly informative, visual, structured and natural in terms of human perception. An image is mixed of initial (non-processed, "real") data, their representations and some deformations. Representations show the information and physical nature of objects, events and processes reproduced by the image while deformations are due to technical characteristics of the tools used to register, form and transform the image in the course of constructing the hierarchy of representations. Thus, when developing methods of formal description of images, in addition to values of brightness of pixels of the image, we need to take into account the extra information associated with it explicitly and implicitly.

It appears that, perceiving images, people do not construct any verbal description but treat images as some integral pattern or a system of such patterns, using a non-linguistic internal representation [29, 30, 38]. When developing methods and systems of automatic image recognition, we have to search for efficient ways to formalize images so that we could work with representations (descriptions) that show the image context and semantics, information hidden in its internal structure and structure of external bonds of part of the real world (scene) reproduced by the image.

In the course of recognition (i.e., when the image is formalized as the recognition object), three types of information characterize the "content" of the image: (a) identifiable objects with well-defined structure, (b) objects with ill-defined structure, and (c) non-identifiable objects.

Thus, to recognize the image, we should use information that show the way the "pattern", i.e., both the image as a whole and the objects represented on it, was formed. To take the image structure into account, we need to determine sub-images—objects—that can be singled out on and see whether they can or must be primitive and what are the relations between these objects and elements. Hence, to use structural information to describe recognition objects, we need to study and use structures of relations between elements, their sets and configurations that form the image. To implement this method, we construct the image model in the form of a hierarchic structure of simpler objects. As a result, we can represent and use hierarchic structural information included in the image explicitly—the image is described using a system of objects, each object is described by simpler objects, the latter are described by simpler objects, and so on.

There are two ways to introduce structural information into the recognition process.

Firstly, similar to the classical recognition problem, we can use the attribute description as the main formalization method, with:

(a) the description having such attributes that characterize interconnection (relations) of individual attributes and/or their groups;

(b) the very attributes being assigned weights, indicating the degree of their importance for object description; and

(c) individual attributes being combined in sets and treated as one attribute.

The second way to introduce structural information in the recognition process leverages the fact that structural information is regular, which is intrinsic to the real world and results in different regularities and struc-

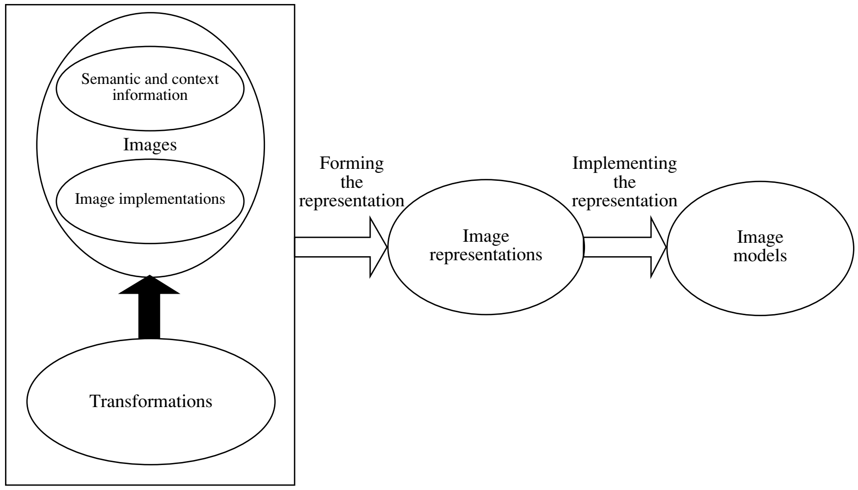


Fig. 4. The scheme to construct image models.

tures. To describe the very image (as well as the complex object on the image), we construct the hierarchic structure formed by simpler objects, which allows us to use and represent structural information included in the image explicitly. Combinatorial regular structures that, in addition, allow taking into account the hierarchy nature of images as recognition objects serve as the mathematical analogue of this idea [7, 8]. Operating with combinatorial regular structures, with a rather limited number of primitive elements and limited set of combination rules applied unrestrictedly to initial primitive elements and results of applying individual combinations of transformations yields almost unbounded diversity of descriptions.

This principle of describing images within the DAIA is called the "generation principle." It is based on the generalized inductive definition (to give some class of objects, we list some initial (generally, basic) objects and rules that allow obtaining new objects of the class to be specified from certain objects) and the concept of combinatorial regular structures (Grenander [7, 8]).

To formalize image description and its conceptual structure, it is natural to assume that the initial image is given not only by its digital implementations but by also context and semantic information that shows the ways of obtaining and forming the image or some its specific aspects.

To construct formalized image descriptions, we need to apply transformations from the set of transformations admissible for this type of images to all information available on the image. Thus, we need to study, first, types of information contained in the image (the space of initial data) and, second, transformations that can be applied to initial images to reduce them to the form supported by recognition algorithms (the space of transformations).

Descriptions of the ways of sequential or/and parallel application of transformations from the space of transformations to the initial information from the space of initial data form the set of schemes to construct formal descriptions of images (the space of image representations).

To be able to apply recognition algorithms to the obtained formal image descriptions, we need to implement the constructed schemes (implement image representations), i.e., construct image models that result from reducing the initial image (taking into account all information on the image) to the form supported by recognition algorithms, i.e., to the easy-to-recognize form. The space of image representations is an intermediate space between the space of initial data and the space of image models.

Thus, construction of image models involves synthesizing objects from the sets of:

initial data—images;

image transformations that reduce images to the recognizable form;

image representations, viz. schemes of constructing formal image descriptions; and

image models.

Figure 4 represents the scheme to construct image models.

The scheme illustrates the process of synthesizing image models by applying transformations that transfer images from the initial to the final set. Each image from the set of images is described by two subsets that represent the images (their implementations), semantic information they carry and the corresponding context information.

2.2. *Image Representation Hierarchy*

The DAIA deals with three classes of admissible image transformations, viz. procedural transformations, parametric transformations, and generating trans-

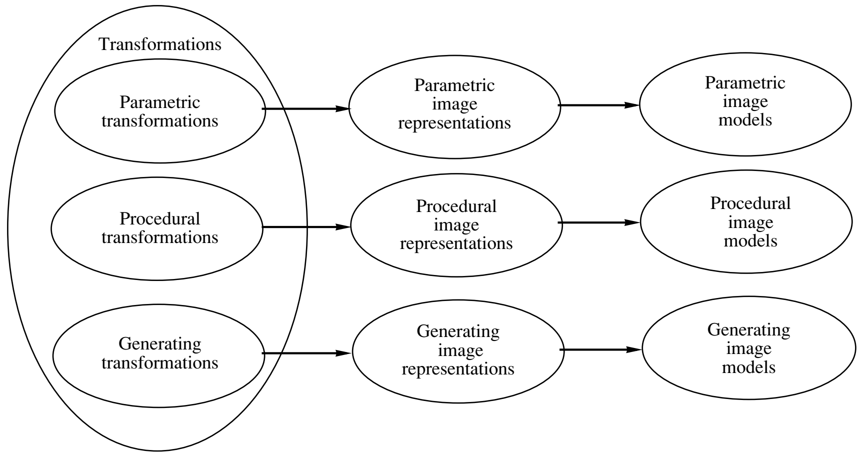


Fig. 5. Relations between classes of transformations, classes of representations and classes of image models.

formations (see definitions 3.2, 3.3, 3.4) [19]. These classes generate three classes of representations and three classes of image models (see Fig. 5).

To characterize images, the DAIA uses the following concepts: the initial information (the image together with its legend), its transformations, representations (a representation is a formal scheme for describing the image and objects it holds) and models (an image model is an image description supported by recognition algorithms).

To determine types of representations undergone by the initial image in the course of constructing its descriptive model and establishing the relations between these types, we introduce additional objects. These are structuring elements, generating rules, semantic and context information on the image, digital implementations of images, classes of image representations, implementation of the image representation, classes of image models, and the correct image model.

Studying the main ideas to construct image descriptions yielded the following relations between objects:

(1) deterministic (obvious) relations between the initial information (the image):

transformations applied to it,

ways of its digital implementations,

results of transforming digital implementations of the initial information;

(2) the DAIA-imposed relations,

between classes of transformations of the initial information and classes of its possible representations,

between classes of possible representations of the initial information and classes of image descriptions in the form supported by recognition algorithms (classes of image models);

(3) special relations obtained:

between some classes of image models,

between some class of image models and the initial information,

between the results of transforming digital implementations of the initial information and classes of image models.

Having studied these relations, we constructed the hierarchy of the DAIA concepts. With the hierarchic scheme given in Fig. 6, we can structure the introduced concepts in order to form algorithmic schemes of solving the image analysis and recognition problem and use DIA to describe images. This hierarchy has also allowed us to state several DAIA axioms [19].

The constructed scheme shows several levels of relations between the DAIA concepts.

(1) In Fig. 6, the two-arrowed double line stands for the relation when:

object 1 is put into correspondence with objects 2, 3, …;

initial image I is put into correspondence with three sets: (a) the set of transformations $\{\tilde{O}\}$ (the set of structuring elements $\{\tilde{S}\}$ is an auxiliary set for the set of transformations $\{\tilde{O}\}$), (b) the set of initial information $\{\tilde{I}_0\}$; (c) the set of image models $\{\tilde{M}\}$.

(2) The set of transformations $\{\tilde{O}\}$ applied to the set of initial information $\{\tilde{I}_0\}$ yields the set of correct image models $\{\tilde{M}\}$ (which is proved in theorem 3.1).

(3) The solid thick line stands for the relation "Object 1 consists of object 2, object 3, ….

The set of transformations $\{\tilde{O}\}$ consists of three sets of transformations: (a) procedural transformations $\{\tilde{O}_T\}$, (b) parametric transformations $\{\tilde{O}_P\}$, and (c) generating transformations $\{\tilde{O}_G\}$. The set of transformations $\{\tilde{O}\}$ (along with three subsets of transformations $\{\tilde{O}_T\}$, $\{\tilde{O}_P\}$, $\{\tilde{O}_G\}$) is given together with the set of structuring elements $\{\tilde{S}\}$, which can be applied

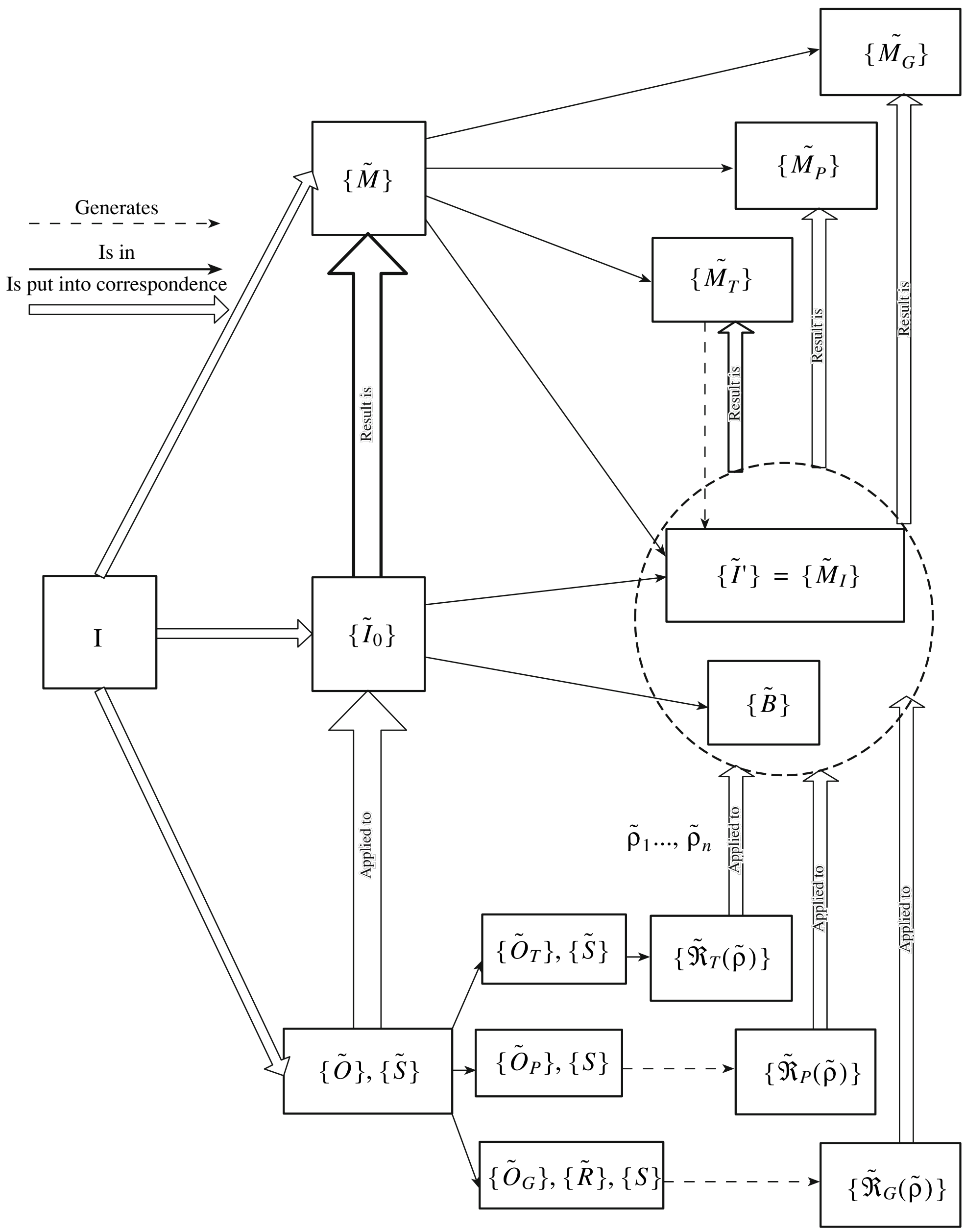


Fig. 6. The DAIA concepts hierarchy.

to the image along with transformations. The set of generating rules $\{\tilde{R}\}$ describe the rules to apply generating transformations $\{\tilde{O}_G\}$ to the initial information $\{\tilde{I}_0\}$. We can say that some subset of generating transformations $\{\tilde{O}_G\}$ is put into correspondence with some fixed subset of generating rules $\{\tilde{R}\}$.

The set of image models $\{\tilde{M}\}$ consists of four sets of models: (a) procedural image models $\{\tilde{M}_T\}$, (b) parametric image models $\{\tilde{M}_P\}$, (c) generating image models $\{\tilde{M}_G\}$, and (d) *I*-models of images $\{\tilde{M}_I\}$.

The initial information $\{\tilde{I}_0\}$ includes both context and semantic information on the image $\{B\}$ as well as the set of implementations $I' \in \{\tilde{I}'\}$ of the image *I* that represent the given object and scene.

(4) The dotted line stands for the relation "The object generates another object."

Three classes of transformations $\{\tilde{O}_T\}$, $\{\tilde{O}_P\}$, $\{\tilde{O}_G\}$ generate three classes of image representations, viz. procedural representations $\{\tilde{\Re}_T(\tilde{\rho})\}$, parametric representations $\{\tilde{\Re}_P(\tilde{\rho})\}$, and generating representations $\{\tilde{\Re}_G(\tilde{\rho})\}$.

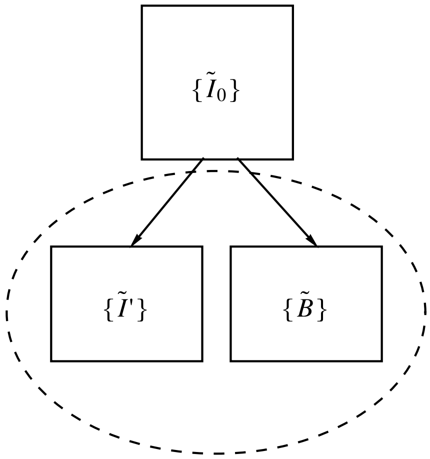


Fig. 7. The set of the initial information.

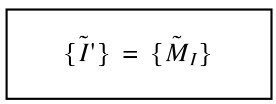


Fig. 8. The identity of *I*-models of images and image implementations.

Any *T*-model of the image $M_T \in \{\tilde{M}_T\}$ is proved to generate some implementation of the image $I' \in \{\tilde{M}_T\}$ (see proposition 3.2).

(5) The dotted oval in the scheme emphasizes the relation between implementations of images $\{\tilde{I}'\}$ and semantic and context information $\{B\}$, which means that different types of initial information are used in the process of solving the problem.

3. DESCRIPTIVE IMAGE MODELS

3.1. Initial Information

The DAIA assumes [19] that the image is described by the set of initial information $\{\tilde{I}_0\}$. We determine the structure of this set.

Lemma 3.1. The set $\{\tilde{I}_0\}$ of initial data consists of two subsets $\{\tilde{I}'\}$ and $\{\tilde{B}\}$: (1) the set of realizations $I' \in \{\tilde{I}'\}$ of I representing the given object or scene such that $I' = \{(x, f(x))\}_{x \in D_f}$ is the set of points x lying in the domain D_f of the image realization and the set of values $f(x)$ at each point of D_f; and (2) semantic and contextual information $\{\tilde{B}\}$ on the image.

The definitional domain of the image implementation is a subset of the n-dimensional discrete space Z^n. For the case of plane (two-dimensional) images, $n = 2$.

Figure 7 illustrates Lemma 3.1.

Definition 3.1. An *I*-model of an image is any element I' of a set $\{\tilde{I}'\}$ of image realizations.

Figure 8 illustrates the relation between the set of *I*-models of images and the set of image implementations.

3.2. Transformations over Images

We consider the set of transformations $\{\tilde{O}\}$ introduced over information given in the form of images.

In this section, we determine the main classes of image transformations (procedural, parametric, and generating) and the related concepts of structuring element, generating rule and correct generating transformation.

Definition 3.2. The *procedural transformation* $O_T \in \{\tilde{O}_T\}$ of the arity r over the images $\{I'_i\}_{1\dots r}$ is an operation that, when applied to the set of images $\{I'_i\}_{1\dots r}$, transforms it into some other set of images, some image or its fragments.

In this case, the procedural transformation $O_T \in \{\tilde{O}_T\}$ of the arity r over *I*-models of images $\{I'_i\}_{1\dots r}$ is an operation that, when applied to the set of *I*-models of images $\{I'_i\}_{1\dots r}$, transforms it into some other set of *I*-models of images, some *I*-model of the image or the set of *I*-models of image fragments. Both *I*-models of one initial image and *I*-models of different initial images can act as operands of this operation.

Definition 3.3. The *parametric transformation* $O_P \in \{\tilde{O}_P\}$ over the image I is an operation that, when applied to the image I, transforms it into the numerical characteristic p, which can be put into correspondence with properties of geometrical objects, brightness characteristics or structures formed when geometrical objects and brightness characteristics of the initial image are repeated regularly.

To construct the numerical characteristic p of the image I, we can use both the set of image implementations and semantic or context information on the image.

Definition 3.4. The *generating transformation* $O_G \in \{\tilde{O}_G\}$ over the image I is an operation that, when applied to the image I, transforms it into some particular representation that shows specific features of the analyzed image.

For the definition of representation, see definition 3.8.

Functions that describe curves, conjunction functions, disjunction functions, and code functions for images can act as examples of such transformations.

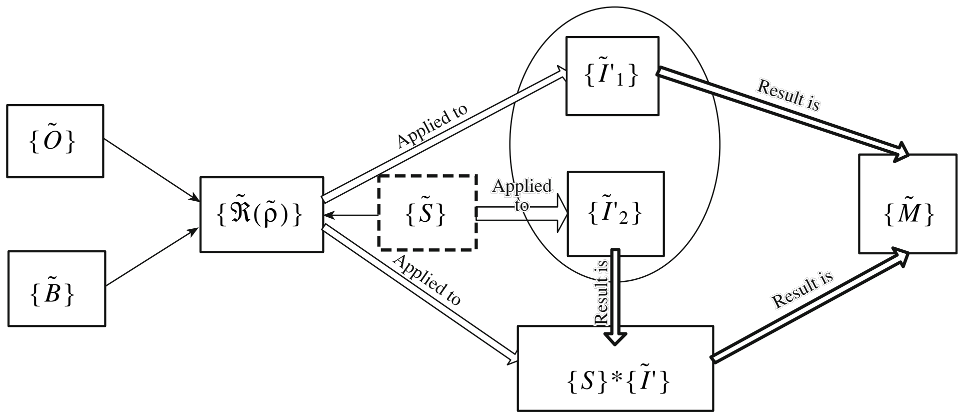


Fig. 9. Image model construction.

Definition 3.5. *A generating rule R* for constructing an image model that determines a strict sequence of generating transformations applied to the image in order to construct its model.

For the definition of image model, see definition 3.9.

Definition 3.6. *A generating transformation O_G* is correct for the given image if and only if there exist generating rules, according to which this generating rule O_G allows constructing the generating image model.

Note that to construct the generating image model (*G*-model of the image), we implement the image generating representation (*G*-representation). For the definition of generating representation and its implementation, see definitions 3.17 and 3.18.

Definition 3.7. *A structuring element* $S \in \{\tilde{S}\}$ is a two-dimensional spatial object that when convoluted with the image divides it into the system of neighborhoods to analyze it locally. A structuring element is specified by parameters defining its form and numerical and geometrical characteristics.

3.3. Image Representations and Models

Definition 3.8. *An image representation* $\Re(I)$ is a formal scheme designed to obtain a standardized formal description of surfaces, point configurations, shapes that form the image and relations between them.

Definition 3.9. *An image model M*(*I*) is a formal description constructed by implementing an image representation $\Re(I)$.

Definition 3.10. *A realization of an image representation* is the application of the representation to realizations of the original image with particular parameter values specified for the transformations involved in the representation.

Definition 3.11. *A correct representation of the image I* is an element of the set of image representations constructed using context and semantic information $\{\tilde{B}\}$ by transformations $\{\tilde{O}\}$ and structuring elements $\{\tilde{S}\}$, where the sets $\{\tilde{B}\}$, $\{\tilde{O}\}$, $\{\tilde{S}\}$ are put into correspondence with the initial image *I*; the set $\{\tilde{S}\}$ can be empty.

Definition 3.12. *A correct image model* is an element of a set of image models generated by implementing correct image representations on the set of initial data $\{\tilde{I}_0\}$.

Theorem 3.1. Any element *m* of the set $\{\tilde{M}\}$ obtained by applying transformations from the set $\{\tilde{O}\}$ to the set of initial data $\{\tilde{I}_0\}$ is *a correct model of the image I*.

Proof. As a result of applying different sequences of transformations from the set $\{\tilde{O}\}$ to the set of initial data $\{\tilde{I}_0\}$, different formal schemes of describing shapes, surfaces and point configurations that form the image and relations between them are generated, i.e., by definition 3.8, the set of image representations is generated. By definition 3.11, any element of this set of image representations is the correct representation of the image *I* since it is constructed using sets of transformations and the initial information that corresponds to the given image. When we give parameters of transformations of any of constructed image representations and implement it on the set of initial data $\{\tilde{I}_0\}$, the corresponding representation is transformed into the set of image models. By definition 3.12, the constructed set includes nothing but the correct image models. The theorem is proved.

Corollary. When transformations from the set $\{\tilde{O}\}$ are applied to the set of initial data $\{\tilde{I}_0\}$, using structuring elements from the set $\{\tilde{S}\}$ that corresponds to the image *I*, the result stays within the set of correct models of the image *I*.

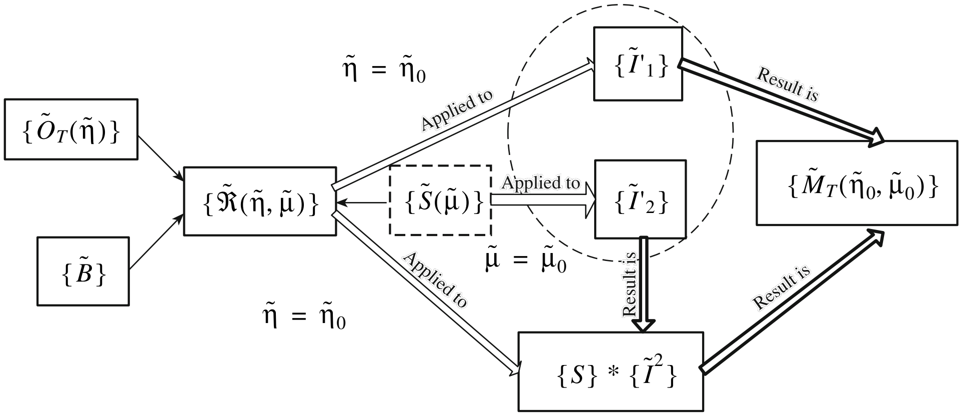


Fig. 10. Generation of the *T*-models of the image.

This corollary can be proved using definitions 3.11 and 3.12.

The reason we introduce all transformations applied to the image model or the image is one of the following:

to obtain the new model;

to reduce the image to the easy-to-recognize form;

to obtain the aggregated estimate of the model, i.e., transfer from the space of initial information to the space of estimates, where procedures of classification decision-making in recognition are implemented.

Schemes (3.1) and (3.2) illustrate relations between image representations and models.

$$\{\tilde{\Re}(I)\} = \{\tilde{O}, \tilde{S}\}: \{I\} \Rightarrow \{\tilde{M}\}, \tag{3.1}$$

$$\{\Re(I)\}(p) = \{O_1, O_2, \ldots, O_n, \tilde{S}'\}(p): \{I'\} \in \{I\} \Rightarrow M_1 \in \{\tilde{M}\}. \tag{3.2}$$

Definition 3.13. *A T-representation* $\Re_T(\tilde{\eta}, \tilde{\mu})$ of the image I is a formal scheme designed to obtain the standardized formal description of the image and constructed using context and semantic information $\{B\} \subset \{\tilde{B}\}$, procedural transformations $\{O_T(\tilde{\eta})\} \subset \{\tilde{O}_T\}$ and structuring elements $\{S(\tilde{\mu})\} \subset \{\tilde{S}\}$ ($\tilde{\eta}$, $\tilde{\mu}$ are the parameters of procedural transformations and structuring elements, respectively).

We denote the set of all correct *T*-representations by $\{\tilde{\Re}_T(\tilde{\eta}, \tilde{\mu})\}$.

Scheme 3.3 illustrates definition 3.13.

$$\{\{\tilde{S}\}, \{\tilde{O}\}\} \xrightarrow{\{B\}} \{\{S(\tilde{\mu})\}, \{O_T(\tilde{\eta})\}\} \longrightarrow \Re_T(\tilde{\eta}, \tilde{\mu}). \tag{3.3}$$

Definition 3.14. *A realization of the T-representation* $\Re_T(\tilde{\eta}, \tilde{\mu})$ of the image I is a process of applying the representation $\Re_T(\tilde{\eta}_0, \bar{\mu}_0)$ with chosen values ($\tilde{\eta} = \tilde{\eta}_0$, $\tilde{\mu} = \tilde{\mu}_0$) of parameters of transformations involved in the representation to implementations of the initial image $\{I'\} \subset \{\tilde{I}'\}$.

Scheme 3.4 illustrates definition 3.14.

$$\Re_T(\tilde{\eta}_0, \tilde{\mu}_0) * \{I'\} = \{S(\tilde{\eta}_0), O_T(\tilde{\mu}_0)\} * \{I'\}. \tag{3.4}$$

Proposition 3.1. Giving the values of parameters of procedural transformations $\tilde{\eta} = \tilde{\eta}_0$ and structuring elements $\tilde{\mu} = \tilde{\mu}_0$, we ensure that any *T*-representation $\Re_T(\tilde{\eta}, \tilde{\mu}) \in \{\tilde{\Re}_T(\tilde{\eta}, \tilde{\mu})\}$ generates the set of *T*-models of the image $\{M_T(\tilde{\eta}_0, \tilde{\mu}_0)\}$.

We denote the set of all correct *T*-models of the image by $\{\tilde{M}_T\}$. Figure 10 illustrates the way *T*-models of images are generated.

Proposition 3.2. Any *T*-model of the image $M_T \in \{\tilde{M}_T\}$ generates some implementation of the image I', i.e., the *I*-model of the image $M_I \equiv I' \in \{\tilde{M}_T\} \equiv \{\tilde{I}'\}$.

Figure 11 illustrates proposition 3.2.

We give an example of the *T*-representation of the image and the set of *T*-models of the image.

Let $\{I'\}$ be the set of implementations of the image I. For a three-dimensional image, it can consist of multiple digital implementations $\{I'\}$ while a two-dimensional image can be described by a set of fragments. Let $\{\tilde{O}_T\} = \{O_j(\tilde{\eta}_j)\}_{1\ldots r}$ be the set of transformations over $\{I'\}$ and $\tilde{\eta}_j$ be the parameters of the procedural transformation O_j. Using the context and semantic information, we specify the sequence of transformations $\{\tilde{O}_T\} = \{O_j(\tilde{\eta}_j)\}_{1\ldots r}$ to be applied, which is the *T*-representation of the image. In this case, we apply

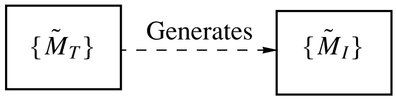


Fig. 11. Generation of the *I*-model by constructing the *T*-model of images.

procedural transformations that do not use structuring elements, i.e., the set $\{S\} = \varnothing$. When the parameters of procedural transformations are given, this *T*-representation of the image, being implemented on the set of implementations $\{I'\}$, generates the set of *T*-models of the image $M_T(I) = \{O_j(\tilde{\eta}_j^0)_{1\ldots r}(\{I'\})$ – operations are applied to image implementations on a parallel or sequential basis.

Definition 3.15. *A P-representation* $\Re_P(\tilde{\eta}, \tilde{\mu})$ of the image I is a formal scheme designed to obtain the standardized formal description of the image and constructed, using the context and semantic information $\{B\} \subset \{\tilde{B}\}$, parametric transformations $\{O_P(\tilde{\eta})\} \subset \{\tilde{O}_P\}$ and structuring elements $\{S(\tilde{\mu})\} \subset \{\tilde{S}\}$ ($\tilde{\eta}$, $\tilde{\mu}$ are the parameters of parametric transformations and structuring elements, respectively).

We denote the set of all correct *P*-representations by $\{\tilde{\Re}_P(\tilde{\eta}, \tilde{\mu})\}$. Scheme 3.5 illustrates definition 3.15.

$$\{\{\tilde{S}\}, \{\tilde{O}\}\} \xrightarrow{\{B\}} \{\{S(\tilde{\mu})\}, \{O_P(\tilde{\eta})\}\} \longrightarrow \Re_P(\tilde{\eta}, \tilde{\mu}). \quad (3.5)$$

Definition 3.16. *A realization of the P-representation* $\Re_P(\tilde{\eta}, \tilde{\mu})$ of the image I is a process of applying the representation $\Re_P(\tilde{\eta}_0, \tilde{\mu}_0)$ with chosen values ($\tilde{\eta} = \tilde{\eta}_0$, $\tilde{\mu} = \tilde{\mu}_0$) of parameters of transformations involved in the representation to implementations of the initial image $\{I'\} \subset \{\tilde{I}'\}$.

Scheme 3.6 illustrates definition 3.16.

$$\Re_P(\tilde{\eta}_0, \tilde{\mu}_0) * \{I'\} = \{S(\tilde{\eta}_0), O_P(\tilde{\mu}_0)\} * \{I'\}. \quad (3.6)$$

Proposition 3.3. If the values of parameters of parametric transformations $\tilde{\eta} = \tilde{\eta}_0$ and structuring elements $\tilde{\mu} = \tilde{\mu}_0$ are given, any *P*-representation $\{M_P(\tilde{\eta}_0, \tilde{\mu}_0)\}$ generates the set of *P-models of the image* $\Re_P(\tilde{\eta}, \tilde{\mu}) \in \{\tilde{\Re}_P(\tilde{\eta}, \tilde{\mu})\}$.

We denote the set of all correct *P*-models of the image by $\{\tilde{M}_P\}$. Figure 12 illustrates the way *P*-models of the image are generated.

We give an example of the *P*-representation and the set of *P*-models.

Let I' and I'' be *I*-models of the initial image I. Thus, a color image can be stored in RGB format (I') or as continuous-tone image (I''). Let $\{O_P\} = \{f_1(\tilde{\eta}_1), f_2(\tilde{\eta}_2), \ldots, f_n(\tilde{\eta}_n)\}$, where $f_1, f_2, \ldots, f_{n_1}$ are the functions that calculate attributes on the *I*-model of the image $I'' \subset \{\tilde{I}'\}$, $f_{n_1+1}, f_{n_1+2}, \ldots, f_n$ are the functions that calculate attributes on the *I*-model of the image $I'' \subset \{\tilde{I}'\}$, and $\tilde{\eta}_1, \tilde{\eta}_2, \ldots, \tilde{\eta}_n$ are parameters of the functions that calculate attributes. To choose functions that calculate attributes, we use context and semantic information on the image. The set of functions that calculate attributes of the image $\{f_1(\tilde{\eta}_1), f_2(\tilde{\eta}_2), \ldots, f_n(\tilde{\eta}_n)\}$ is the *P*-representation of the image. In this case, parametric transformations are applied that do not use structuring elements, i.e., the set $\{S\} = \varnothing$.

When the parameters of parametric transformations are given, this *P*-representation of the image, being implemented on the set of implementations I' and I'', generates the set of *P*-models of the image I with the given parameters – the vector of numerical attributes

$$M_P(I) = (f_1(\tilde{\eta}_1^0)(I'), f_2(\tilde{\eta}_2^0)(I'),$$

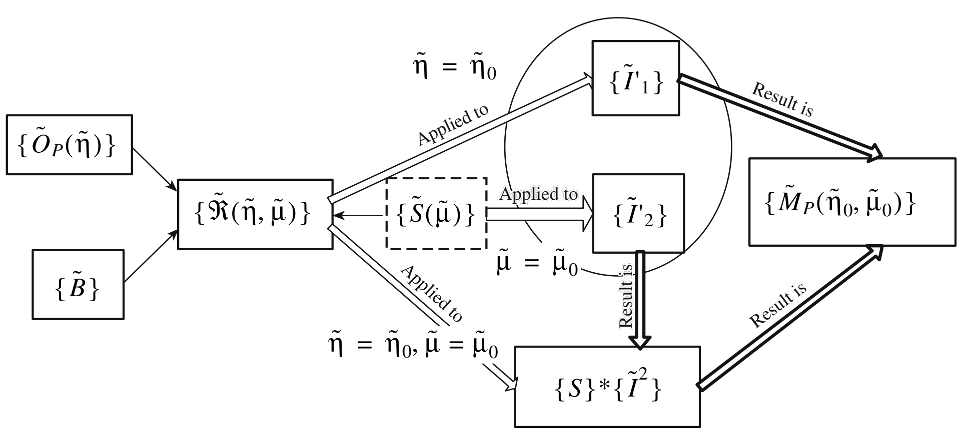


Fig. 12. Generation of *P*-models of the image.

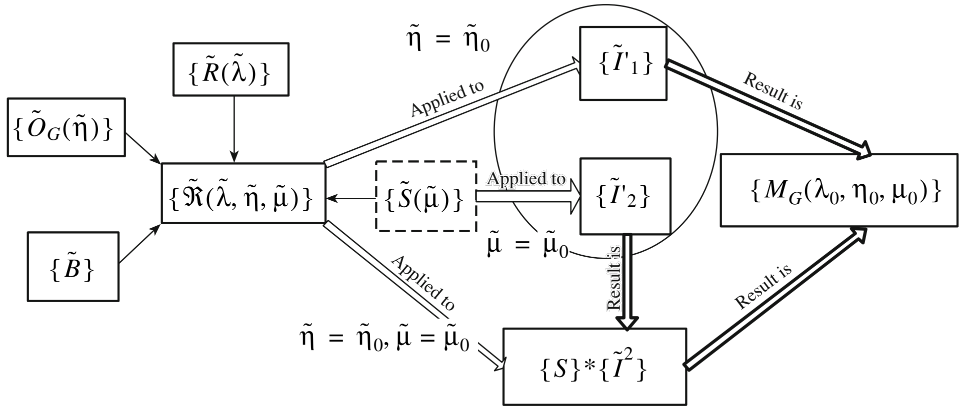


Fig. 13. Generation of *G*-models of the image.

$$\ldots, f_{n_1}(\tilde{\eta}^0_{n_1})(I'), f_{n_1+1}(\tilde{\eta}^0_{n_1+1})(I''),$$

$$f_{n_1+2}(\tilde{\eta}^0_{n_1+2})(I''), \ldots, f_n(\tilde{\eta}^0_n)(I'')).$$

Definition 3.17. *A G-representation* $\Re_G(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu})$ of an image *I* is a formal scheme designed to obtain a standardized formal description of the image and constructed using context and semantic information $\{B\} \subset \{\tilde{B}\}$ by the generating rules $\{\tilde{R}(\tilde{\lambda})\}$ that completely give the sequence of generating transformations $\{O_G(\tilde{\eta})\} \subset \{\tilde{O}_G\}$ and structuring elements $\{S(\tilde{\mu})\} \subset \{\tilde{S}\}$ to be applied to the image ($\tilde{\lambda}$, $\tilde{\eta}$, and $\tilde{\mu}$ are the parameters of generating rules that generate transformations and structuring elements, respectively).

We denote the set of all correct *G*-representations by $\{\tilde{\Re}_G(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu})\}$. Scheme 3.7 illustrates definition 3.17.

$$\{\{\tilde{R}\}, \{\tilde{S}\}, \{\tilde{O}_G\}\}$$
$$\xrightarrow{\{B\}} \{\{R(\tilde{\lambda})\}, \{S(\tilde{\mu})\}, \{O_P(\tilde{\eta})\}\} \longrightarrow \Re_G(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu}). \tag{3.7}$$

Definition 3.18. *A realization of the G-representation* $\Re_G(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu})$ of a image *I* is a process of applying of the representation $\Re_P(\tilde{\lambda}_0, \tilde{\eta}_0, \tilde{\mu}_0)$ with chosen values ($\tilde{\lambda} = \tilde{\lambda}_0$, $\tilde{\eta} = \tilde{\eta}_0$, $\tilde{\mu} = \tilde{\mu}_0$) of parameters of transformations involved in the representation to implementations of the initial image $\{I'\} \subset \{\tilde{I}'\}$.

Scheme 3.8 illustrates definition 3.18.

$$\Re_G(\tilde{\lambda}_0, \tilde{\eta}_0, \tilde{\mu}_0) * \{I'\}$$
$$= \{R(\tilde{\lambda}_0), S(\tilde{\eta}_0), O_G(\tilde{\mu}_0)\} * \{I'\}. \tag{3.8}$$

Proposition 3.4. If the values of parameters of generating rules $\tilde{\lambda} = \tilde{\lambda}_0$, generating transformations $\tilde{\eta} = \tilde{\eta}_0$ and structuring elements $\tilde{\mu} = \tilde{\mu}_0$ are given, any *G*-representation $\{M_G(\tilde{\lambda}_0, \tilde{\eta}_0, \tilde{\mu}_0)\}$ generates the set of *G-models of the image* $\Re_G(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu}) \in \{\tilde{\Re}_G(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu})\}$.

We denote the set of all correct *G*-models of the image by $\{\tilde{M}_G\}$. Figure 13 illustrates the way *G*-models of the image are generated.

Figure 6 shows interconnections between concepts given in Section 3.

CONCLUSIONS

At present, automated image mining is the main strategic goal of fundamental research in image analysis, recognition and understanding and development of the proper information technology and algorithmic software systems. In the long run, this automatization is expected to help developers of automated systems designed to handle images as well as end users, either in the automated or interactive mode:

—develop, adapt and check methods and algorithms of image recognition, understanding and evaluation;

—choose optimal or suitable methods and algorithms of image recognition, understanding and evaluation;

—check the quality of initial data and whether they can be used in solving the image recognition problem; and

—apply standard algorithmic schemes of image recognition, understanding, evaluation and search.

To ensure such automatization, we need to develop and evolve a new approach to analyzing and evaluating information represented in the form of images. To do it, the "Algebraic approach" of Yu. I. Zhuravlev was modified for the case when the initial information is represented in the form of images. The result is the DAIA.

By now, image analysis and evaluation have a wide experience gained in applying mathematical methods from different sections of mathematics, computer science and physics, in particular algebra, geometry, discrete mathematics, mathematical logic, probability theory, mathematical statistics, mathematical analysis, mathematical theory of pattern recognition, digital signal processing, and optics.

On the other hand, with all this diversity of applied methods, we still need to have a regular basis to arrange and choose suitable methods of image analysis, represent, in an unified way, the processed data (images), meeting the requirements standard recognition algorithms impose on initial information, construct mathematical models of images designed for recognition problems, and, on the whole, establish the universal language for unified description of images and transformations over them.

Proposed and being developed as a conceptual and logical basis for image analysis and recognition, the DAIA embraces a totality of methods of image analysis and recognition, methods of reducing images to an easy-to-recognize form, the system of concepts of image analysis and recognition, classes of descriptive image models (DIM), statements of image analysis and recognition problems, and the basic model of the image recognition process.

The main DAIA objects and tools are images, the universal language, viz. DIA, and models of two types: models of images and models of procedures to solve image recognition problems and their implementations in the form of algorithmic schemes.

When we move from classical recognition problems to image recognition problems, there arise mathematical problems due to formal description of the image as the object to be analyzed.

On the whole, whether analysis and evaluation of information represented in the form of images is successful and efficient depends on IRRF capabilities. IRRF processes are critical in solving applied problems of image analysis and, in particular, in intellectual decision-making based on image mining.

We can leverage the DAIA to solve both problems of constructing formal descriptions of images as recognition objects and problems of synthesizing image recognition and understanding procedures.

The main contribution of this work is that we form the system of concepts that characterize the initial information, viz. images, in recognition problems and set unambiguously the hierarchic system of relations introduced on the classes of these concepts.

Being developed as the fundamental basis of the mathematical theory of image analysis and recognition, the DAIA allows introducing the axiomatics of the sought theory.

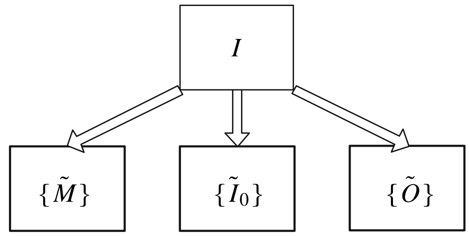


Fig. 14. Illustration of axiom 1.

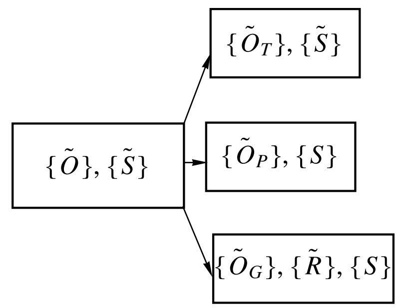


Fig. 15. Illustration of axiom 2.

The system of concepts and the formal apparatus of descriptive image models introduced form the necessary background to state the main axioms of the mathematical theory of image analysis and recognition. We give the following axioms as an example.

Axiom 1. *Any image I* can be unambiguously put into correspondence with the totality of sets ($\{\tilde{I}_0\}$, $\{\tilde{O}\}$, $\{\tilde{M}\}$), where $\{\tilde{I}_0\}$ is the set of initial information, $\{\tilde{O}\}$ is the set of transformations applicable to the set of initial information, and $\{\tilde{M}\}$ is the set of results of applying transformations to the initial information.

The following scheme and Fig. 14 illustrate axiom 1.

$$\{O\}: (I \cong \{\tilde{I}_0\}) \Rightarrow \{\tilde{M}\}.$$

Axiom 2. The set of transformations $\{\tilde{O}\}$ is given by the set of structuring elements $\{\tilde{S}\}$, the set of generating rules $\{\tilde{R}\}$ and three subsets of transformations $\{\tilde{O}_T\}$, $\{\tilde{O}_P\}$, $\{\tilde{O}_G\}$—(1) procedural transformations $\{\tilde{O}_T\}$, (2) parametric transformations $\{\tilde{O}_P\}$, and (3) generating transformations $\{\tilde{O}_G\}$.

Figure 15 illustrates axiom 2.

To be continued…

ACKNOWLEDGMENTS

This study was partially supported by the Russian Foundation for Basic Research (grants no. 08-01-00469, 07-07-13545, 06-07-89203); the mathematical sciences division of the Russian Academy of Sciences ("Descriptive Image Algebras with One Ring over Image Models" project within the "Algebraic and Combinatorial Methods of Mathematical Cybernetics" program for fundamental research), and the Russian Academy of Sciences presidium (project no. 2.14 within the "Fundamental Problems of Computer Science and Information Technology" program).

REFERENCES

1. H. G. Barrow, A. P. Ambler, and R. M. Burstall, "Some Techniques for Recognizing Structures in Pictures," in *Frontiers of Pattern Recognition, Proc. of the International Conference on Frontiers of Pattern Recognition*, Ed. by Satosi Watanabe (Academic Press, New York, London, 1972), pp. 1–30.
2. M. B. Clowes, "Perception, Picture Processing and Computers," Machine Intelligence I, Ed. by Collins and D. Michie, (Oliver and Boyd, London, 1967).
3. M. B. Clowes, "Pictorial Relationships—a Syntactic Approach," Machine Intelligence IV, Ed. by Meltzer and Michie (Edin, Uni Press, 1969).
4. T. G. Evans, "A Formalism for the Description of Complex Objects and Its Implementation," in *Proc. of the Firth International Conference on Cybernetics, Namur, Belgium, September, 1967.*
5. T. G. Evans, "Descriptive Pattern Analysis Techniques: Potentialities and Problems," in *Methodologies of Pattern Recognition, Proc. of the International Conference on Methodologies of Pattern Recognition* (Academic Press, New York, London, 1969), pp. 149–157.
6. A. L. Gorelik, I. B. Gurevich, and V. A. Skripkin, *Modern State of Recognition Problem: Some Aspects* (Radio i Svyaz', Moscow, 1985) [in Russian].
7. U. Grenander, *Lectures in Pattern Theory*, Vol. 1, Chapter 3, 3.1: *Image Algebra* (N.Y.: Sprinder-Verlag, 1976, Vol. l; 1978, Vol. 2; 1981, Vol. 3; Mir, Moscow, 1979).
8. U. Grenander, *General Pattern Theory. A Mathematical Study of Regular Structure* (Clarendon Press, Oxford, 1993).
9. I. B. Gurevich, "The Descriptive Framework for an Image Recognition Problem," in *Proc. of the 6th Scandinavian Conference on Image Analysis* I (Pattern Recognition Society, Finland, 1989), Vol. 1, pp. 220–227.
10. I. B. Gurevich, "Descriptive Technique for Image Description, Representation and Recognition," Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications in the USSR **1**, 50–53 (1991).
11. I. B. Gurevich and V. V. Yashina, "Descriptive Image Algebras with One Ring," Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **13** (4), 579–599 (2003).
12. I. B. Gurevich and I. A. Jernova, "A Way to Characterize Image Equivalence," in *Proc. of the 6th German-Russian Workshop on Pattern Recognition and Image Understanding (OGRW-6-2003), Katun Village, Altai Region, Russian Federation, August 25–30, 2003*, pp. 88–91.
13. I. B. Gurevich and I. A. Jernova, "The Joint Use of Image Equivalents and Image Invariants in Image Recognition," Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **13** (4), 570–578 (2003).
14. I. B. Gurevich, "The Descriptive Approach to Image Analysis. Current State and Prospects," in *Proc. of the 14th Scandinavian Conference on Image Analysis, Springer-Verlag Berlin Heidelberg, 2005, LNCS 3540*, pp. 214–223.
15. I. B. Gurevich and V. V. Yashina, "Generating Descriptive Trees," in *Proc. of the 10th International Fall Workshop on Vision, Modeling, and Visualization* (Infix, 2005), pp. 367–374.
16. I. B. Gurevich and V. V. Yashina, "Operations of Descriptive Image Algebras with One Ring," Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **16** (3), 298–328 (2006).
17. I. B. Gurevich and V. V. Yashina, "Computer-Aided Image Analysis Based on the Concepts of Invariance and Equivalence," Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **16** (4), 564–589 (2006).
18. I. Gurevich, I. Koryabkina, V. Yashina, H. Niemann, and O. Salvetti, "An Application of a Descriptive Image Algebra for Diagnostic Analysis of Cytological Specimens. An Algebraic Model and Experimental Study," in *Proc. of VISAPP 2007—Second International Conference on Computer Vision Theory and Applications Special Sessions* (INSTICC Press, 2007), pp. 230–237.
19. I. Gurevich and V. Yashina, "Descriptive Analysis of Image Data. Basic Models," in *Image Mining Theory and Applications: Proc. of the 1st International Workshop on Image Mining Theory and Applications—IMTA 2008 (in Conjunction with VISIGRAPP 2008), Funchal, Madeira, Portugal, January 2008*, Ed. by I. Gurevich, H. Niemann, and O. Salvetti (INSTICC PRESS, 2008), pp. 3–15.
20. I. Gurevich, V. Yashina, H. Niemann, and O. Salvetti, "Medical Image Mining on the Base of Descriptive Image Algebras. Cytological Specimen Case," in *Proc. of the International Conference on Health Informatics—HEALTHINF 2008, Funchal, Madeira, Portugal, January 28–31, 2008* (INSTICC PRESS, 2008), vol. 2, pp. 66–73.
21. S. Kaneff, "Pattern Cognition and the Organization of Information," in *Frontiers of Pattern Recognition, Proc. of the International Conference on Frontiers of Pattern Recognition*, Ed. by Satosi Watanabe (Academic Press, New York, London, 1972), pp. 193–222.
22. J. Kittler and F. M. Alkoot, "Relationship of Sum and Vote Fusion Strategies," in *Multiple Classifier Systems. Proc. of Second International Workshop, Springer-Verlag, 2001*, pp. 339–348.
23. R. Kirsh, "Computer Interpretation of English Text and Picture Patterns," IEEE-TEC **EC-13** (4) (1964).
24. D. Marr, *Vision* (Freeman, New York, 1982).
25. R. Narasimhan, "Syntax-Directed Interpretation of Classes of Pictures," Community ACM **9** (3) (1966).

26. R. Narasimhan, "Labeling Schemata and Syntactic Descriptions of Pictures," Information and Control **7** (2) (1967).

27. R. Narasimhan, "On the Description, Generalization and Recognition of Classes of Pictures," in *Proc. of NATO Summer School on Automatic Interpretation and Classification of Images, Pisa, Italy, August 26–September 7, 1968*.

28. R. Narasimhan, "Picture Languages," Picture Language Machines, Ed. by S. Kaneff (Academic Press, London, New York, 1970), pp. 1–30.

29. S. E. Palmer, *Vision Science. Photons to Phenomenology* (A Bradford Book, The MIT Press, Cambridge, Massachusetts London, England, 1999).

30. S. Pinker, "Visual Cognition: An Introduction," Visual Cognition, *A Cognition Special Issue*, Ed. by S. Pinker (Netherlands, 1988), pp. 1–65.

31. G. X. Ritter and J. N. Wilson, *Handbook of Computer Vision Algorithms in Image Algebra*, 2nd ed. (CRC Press Inc., 2001).

32. G. X. Ritter, *Image Algebra* (Center for Computer Vision and Visualization, Department of Computer and Information Science and Engineering, University of Florida, Gainesville, 2001).

33. A. Rosenfeld, *Picture Languages. Formal Models for Picture Recognition* (Academic Press, New York, San Francisco, London, 1979).

34. J. Serra, *Image Analysis and Mathematical Morphology* (L., Academic Press 1982).

35. A. Shaw, "A Proposed Language for the Formal Description of Pictures," CGS Memo (Stanford University, February 1967), Vol. 28.

36. A. Shaw, "The Formal Description and Parsing of Pictures," Ph.D. Thesis (Computer Sciences Department, Stanford University, December 1967); Tech. Rept CS94, April 1968.

37. S. R. Sternberg, "Language and Architecture for Parallel Image Processing," in *Proc. of the Conference on Pattern Recognition in Practice, Amsterdam, 1980*.

38. *The Visual Neurosciences*, Ed. by L. M. Chalupa and J. S. Werner (A Bradford Book, The MIT Press, Cambridge, Massachusetts London, England, 2004).

39. S. H. Unger, "A Computer Oriented toward Spatial Problems," in *Proc. of the IRE, 46, 1958*, pp. 1744–1750.

40. Yu. I. Zhuravlev and A. A. Alekcanyan, "On an Approach to Constructing Efficient Recognition Algorithms," Zhurnal Vych. Mat. i Mat. Fiz. **25** (2) (1965).

41. Yu. I. Zhuravlev and V. V. Nukiforov, "Recognition Algorithms Based of Estimate Calculation," Kibernetika, no. 3 (1971).

42. Yu. I. Zhuravlev, "An Algebraic Approach to Recognition and Classification Problems," Problemy Kibernetiki Vol. 33 (Nauka, Moscow, 1978), pp. 5–68.

43. Yu. I. Zhuravlev, "An Algebraic Approach to Recognition and Classification Problems," Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **8**, 59–100 (1998).

44. Yu. I. Zhuravlev and I. B. Gurevich, *Pattern Recognition and Image Analysis*, in *Artificial intelligence*, Vols. 1–3, (Radio i Svyaz', Moscow, 1990), Vol 2: *Models and Methods: A Refeberce Book*, pp. 149–191 [in Russian].

Igor' B. Gurevich. Born 1938. Received his Dr.-Eng. degree (Diploma Engineer, Automatic Control and Electrical Engineering) from Moscow Power Engineering Institute in 1961. Received his doctor's degree (Mathematical Cybernetics) from Moscow Institute of Physics and Technology in 1975. At present, he is the head of department at the Dorodnicyn Computing Center of the Russian Academy of Sciences; an assistant professor at the Computer Science Faculty, Moscow Lomonosov State University. He has worked as an engineer and researcher in industry, medicine, and universities since 1960 till now and in the USSR/Russian Academy of Sciences since 1985. Area of expertise: image analysis, image understanding, mathematical theory of pattern recognition, theoretical computer science, medical informatics, applications of pattern recognition and image analysis techniques in medicine, nondestructive testing, process control, knowledge-based systems. Author of 2 monographs (in co-authorship), 205 papers in peer reviewed journals and proceedings, and 5 patents. He is the Scientific Secretary of the National Committee for Pattern Recognition and Image Analysis of the RAS, member of the IAPR Governing Board (representative from RF), IAPR Fellow. He has been the PI of many R&D projects as part of national and international research programs. Vice Editor-in-Chief of Pattern Recognition and Image Analysis Journal, member of Editorial Boards of several international scientific journals, member of the committees of many international scientific conferences.

Vera V. Yashina. Born 1980. Received her diploma mathematician from Moscow Lomonosov State University in 2002. At present, she is a researcher at the Dorodnicyn Computing Center of the RAS. Scientific expertise: mathematical theory of image analysis, image algebras, descriptive image models and medical informatics. Author of 17 papers in peer reviewed journals, conference and workshop proceedings. She was awarded several times for the best young scientist papers presented at several international conferences—7th International Conference on Pattern Recognition and Image Analysis: New Information Technologies, 7th Open German-Russian Workshop on Pattern Recognition and Image Analysis, 8th International Conference on Pattern Recognition and Image Analysis: New Information Technologies.